PHYS-UA 120 Dynamics Problem Set 13

Due in the "Dynamics" hand-in box before noon on 2014 December 18.

Problem 1:

Integrate a trajectory starting at $t = 0$ at $x = 0.100$, $y = 0.800$, $v_x = -0.050$, $v_y = 0.020$ for times $0 < t < 50$ in the Hénon–Heiles potential given on pp. 368–369 (with $m = 1$). Use a leap-frog integrator (as described in lecture). Note that the textbook equations have bad units; the coordinates (including time) have been implicitly made dimensionless. Ignore that fact while you solve the problem but then figure out (on your own; not for credit) how that is possible. Hand in a plot of the trajectory in the x–y plane plus a plot of the Hamiltonian vs time (to show that your code is not obviously wrong). Over-plot a second integration (using a different color or linestyle) starting identically except with initial $x = 0.101$. Can you see the exponential divergence of trajectories? Hand in your code along with the plots.

Problem 2:

Reproduce Fig. D.2 as well as you can. The lines are *fixed* points (or limit cycles) of the map, so at each value of r you step through, you have to iterate the map to "convergence" and then plot some (say) 32 points. When the limit is a one-cycle, these points will all hit at the same place, when it is a two-cycle, in two places, and so on. Hand in your figure and your code.